

Renormalization Group Running in the Symmetric and the Broken Symmetry Phases of the R_ξ and the \overline{R}_ξ Gauges

Chungku Kim

(Dated: February 6, 2017)

We investigate the renormalization group (RG) running of the effective potential and the pole mass in the broken symmetry phase of the R_ξ and the \overline{R}_ξ gauges which have different RG running for the effective potential in the symmetric phase and show that if the vacuum expectation value (VEV) is expressed as a function of the other parameters of the theory by solving the minimization condition, then the effective potential in the broken symmetry phase in both gauges satisfies the same RG equation as one in the symmetric phase of the \overline{R}_ξ gauge. The pole masses in the broken symmetry phase of both gauges are RG invariant with respect to the RG functions of the symmetric phase and are shown to be the same at one-loop order.

PACS numbers: 11.15.Bt, 12.38.Bx

The R_ξ gauge is widely used in gauge theory with spontaneous symmetry breaking due to the fact that the mixing between the gauge field and the Goldstone boson field in the kinetic term of the Lagrangian is absent in the broken symmetry phase [1]. Because the running mass of the particles in the broken symmetry phase is given by the multiplication of the coupling constants and the vacuum expectation value (VEV), the renormalization group (RG) behavior of the VEV is very important. Recently, the renormalization of the VEV in the R_ξ gauge was investigated and the resulting gamma function of the VEV turned out to be different from that of the scalar field ($\gamma_v \neq \gamma_\phi$) [2,3]. This is the consequence of the fact that the R_ξ gauge has a tadpole divergence in the symmetric phase, and as a result, the scalar field needs both multiplicative and additive renormalization [4,5]. This fact, as well as the fact that the Lagrangian depends on the VEV in the symmetric phase causes a violation of the Higgs-boson low-energy theorem [6]. In order to avoid this problem, the non-linear $R_{\xi/\sigma}$ gauge was investigated [6,7]. Recently, it was shown that if the symmetric phase of the Lagrangian did not contain the VEV, γ_v coincides with γ_ϕ and the identity [8]

$$\xi \frac{\partial v}{\partial \xi} = C_\xi(v), \quad (1)$$

held, where $C_\xi(v)$ was the function given by Nielsen [9] and that if the symmetric phase of the Lagrangian depended on the VEV, which is the case of the R_ξ gauge, this identity should be modified [5]. Moreover, because most RG functions are calculated in the symmetric phase of the MS Lagrangian, many attempts have been made to relate the RG functions in the symmetric phase and those in the broken symmetry phase [10] in different renormalization schemes.

In this paper, we will investigate the RG behavior of the effective potential and the pole mass in the broken symmetry phase of the R_ξ and the \overline{R}_ξ gauges, which is the $\sigma = 1$ case of the $R_{\xi/\sigma}$ gauge. For simplicity, we will consider the case of the Abelian Higgs model with the Lagrangian density

$$\begin{aligned} L_{SYM}(\Phi_1, \Phi_2, A_\mu) = & \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} (\partial_\mu \Phi_1 + g A_\mu \Phi_2)^2 + \frac{1}{2} (\partial_\mu \Phi_2 - g A_\mu \Phi_1)^2 + \frac{1}{2} m^2 (\Phi_1^2 + \Phi_2^2) \\ & + \frac{\lambda}{24} (\Phi_1^2 + \Phi_2^2)^2 + \frac{1}{2\xi} f(\Phi_1, \Phi_2, A_\mu)^2 + \bar{c} \frac{\delta f(A_\mu^\theta, \Phi_2^\theta)}{\delta \theta} c + \text{counter terms}, \end{aligned} \quad (2)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (3)$$

and $f(\Phi_1, \Phi_2, A_\mu)$ is the gauge fixing function.

In the case of RG running for \overline{R}_ξ gauge fixing in the broken symmetry phase, the gauge fixing function is given by

$$f(\Phi_1, \Phi_2, A_\mu) = \partial_\mu A_\mu - \xi g \Phi_1 \Phi_2. \quad (4)$$

Because no tadpole divergence is possible in the symmetric phase, the scalar fields are renormalized multiplicatively as usual, and the corresponding RG equation for the renormalized effective action in the symmetric phase $\Gamma_{SYM}(\{c_i\}, \phi)$ is given as

$$D\Gamma_{SYM}(\{c_i\}, \phi) + \gamma_\phi \phi \frac{\partial \Gamma_{SYM}(\{c_i\}, \phi)}{\partial \phi} = 0, \quad (5)$$

where $\{c_i\}$ is the parameter set containing μ, g, λ, m^2 and ξ , ϕ is the classical field of the scalar field Φ_1 , and the operator D is defined by

$$D = \mu \frac{\partial}{\partial \mu} + \beta_g \frac{\partial}{\partial g} + \beta_\lambda \frac{\partial}{\partial \lambda} + \beta_{m^2} \frac{\partial}{\partial m^2} + \beta_\xi \frac{\partial}{\partial \xi}. \quad (6)$$

Because $\Gamma_{SYM}(\Phi_1, \Phi_2, A_\mu)$ does not depend on the VEV in this gauge, the effective action in the broken symmetry phase $\Gamma_{BS}(\{c_i\}, \phi, v)$ can be obtained as

$$\Gamma_{BS}(\{c_i\}, \phi, v) = \Gamma_{SYM}(\{c_i\}, \phi + v), \quad (7)$$

where the VEV can be obtained from the minimization condition

$$\left[\frac{\partial V_{SYM}(\phi)}{\partial \phi} \right]_{\phi=v} = 0, \quad (8)$$

with the renormalized effective potential in the symmetric phase V_{SYM} being obtained from Γ_{SYM} by taking the classical field ϕ as a constant field. By applying D defined in Eq. (6) to this equation, we can obtain $Dv = \gamma_\phi v$, which means that $\gamma_v = \gamma_\phi$ [5]. Then, by applying D to the effective action in the broken symmetry phase $\Gamma_{BS}(\{c_i\}, \phi, v)$ and by using Eq. (5), we obtain

$$\begin{aligned} D\Gamma_{BS}(\{c_i\}, \phi, v) &= D\Gamma_{SYM}(\{c_i\}, \phi + v) \\ &= -\gamma_\phi(\phi + v) \frac{\partial \Gamma_{SYM}(\{c_i\}, \phi + v)}{\partial \phi} + (Dv) \frac{\partial \Gamma_{SYM}(\{c_i\}, \phi + v)}{\partial v} = -\gamma_\phi \phi \frac{\partial \Gamma_{BS}(\{c_i\}, \phi, v)}{\partial \phi}. \end{aligned} \quad (9)$$

This means that the effective action in the broken symmetry phase $\Gamma_{BS}(\{c_i\}, \phi, v)$ satisfies the same RG equation as that of the effective action in the symmetric phase $\Gamma_{SYM}(\{c_i\}, \phi)$ if we make the substitution

$$v = v(\{c_i\}), \quad (10)$$

as determined from the minimization condition given in Eq. (8) for the VEV in $\Gamma_{BS}(\{c_i\}, \phi, v)$.

In the case of RG running for R_ξ gauge fixing in the broken symmetry phase, the gauge fixing function is given by

$$f(\Phi_1, \Phi_2, A_\mu) = \partial_\mu A_\mu - u \xi g \Phi_2. \quad (11)$$

In this gauge, the tadpole divergence occurs in the symmetric phase[4,5] Hence we need not only the multiplicative but also the additive renormalization for the scalar fields as

$$\phi_B = \sqrt{Z_\phi}(\phi + \frac{1}{2}u \delta \widehat{Z}), \quad (12)$$

which gives

$$\mu \frac{\partial \phi}{\partial \mu} = \gamma_\phi \phi + \widehat{\gamma} u, \quad (13)$$

where $\widehat{\gamma}$ is the $O(\varepsilon^0)$ term of $\frac{1}{2} \mu \frac{\partial \delta \widehat{Z}}{\partial \mu}$. The resulting RG equation in the symmetric phase $\Gamma_{SYM}(\{c_i\}, u, \phi)$ becomes [5]

$$DV_{SYM}(\{c_i\}, u, \phi) + (\gamma_\phi \phi + \widehat{\gamma} u - C_u \gamma_u) \frac{\partial V_{SYM}(\{c_i\}, u, \phi)}{\partial \phi} = 0, \quad (14)$$

where C_u is the function appearing in the Nielsen identity for the gauge parameter u as

$$u \frac{\partial V_{SYM}(\{c_i\}, u, \phi)}{\partial u} + C_u(\phi) \frac{\partial V_{SYM}(\{c_i\}, u, \phi)}{\partial \phi} = 0 \quad (15)$$

and

$$\gamma_u = \frac{\mu}{u} \frac{\partial u}{\partial \mu}. \quad (16)$$

Because the parameter u of the gauge fixing function given in Eq. (11) should be identified as the VEV v in order to remove the mixing term between the gauge field A_μ and the Goldstone field Φ_2 in the kinetic part of the Lagrangian, the effective action in the broken symmetry phase $\Gamma_{BS}(\{c_i\}, \phi, v)$ is obtained from the effective action in the symmetric phase $\Gamma_{SYM}(\{c_i\}, u, \phi)$ as

$$\Gamma_{BS}(\{c_i\}, \phi, v) = [\Gamma_{SYM}(\{c_i\}, u, \phi + v)]_{u=v}. \quad (17)$$

By applying D to the minimization condition for the VEV given in Eq. (8) and by using Eq. (14), we can obtain the RG behavior of VEV as [5]

$$Dv = \frac{(\gamma_\phi + \hat{\gamma})v - C_u(v)\gamma_u}{1 - C_u(v)/v}, \quad (18)$$

and by applying D to the effective action in the broken symmetry phase $\Gamma_{BS}(\{c_i\}, \phi, v)$ and by using Eqs. (14), (15) and (18), we obtain

$$\begin{aligned} D\Gamma_{BS}(\{c_i\}, \phi, v) &= [D\Gamma_{SYM}(\{c_i\}, u, \phi + v)]_{u=v} + (Dv) \left[\frac{\partial V_{SYM}(\{c_i\}, u, v)}{\partial v} + \frac{\partial V_{SYM}(\{c_i\}, u, v)}{\partial u} \right]_{u=v} \\ &= -\gamma_\phi(\phi + v) \left[\frac{\partial \Gamma_{SYM}(\{c_i\}, v, \phi + v)}{\partial \phi} \right]_{u=v} - (\hat{\gamma}v - C_u(v)\gamma_u) [\Gamma_{SYM}(\{c_i\}, v, \phi + v)]_{u=v} \\ &\quad + (Dv) \left(1 - \frac{C_u(v)}{v} \right) \left[\frac{\partial \Gamma_{SYM}(\{c_i\}, u, v)}{\partial v} \right]_{u=v} = -\gamma_\phi \phi \frac{\partial \Gamma_{BS}(\{c_i\}, \phi, v)}{\partial \phi}. \end{aligned} \quad (19)$$

By comparing this equation with that in case of the R_ξ gauge given in Eq. (9), we can see that the RG equation for the effective action in the broken symmetry phase $\Gamma_{BS}(\{c_i\}, \phi, v)$ is the same in both the R_ξ and the \overline{R}_ξ gauges if we substitute $v(\{c_i\})$ determined from the minimization condition given in Eq. (8) for the VEV in $\Gamma_{BS}(\{c_i\}, \phi, v)$ as in the case of the R_ξ gauge.

Now, let us consider the running of the pole mass in the broken symmetry phase in the R_ξ and the \overline{R}_ξ gauges. The pole mass M^2 in the broken symmetry phase is defined as a pole of the two-point Green's function as

$$\left[\frac{\delta^2 \Gamma_{BS}(\{c_i\}, \phi, v)}{\delta \phi^2} \right]_{\phi=0, p^2=-M^2} = 0. \quad (20)$$

By taking the derivative $\frac{\delta}{\delta \phi}$ of the RG equation for the effective action in the broken symmetry phase (Eqs. (9) and (19)) twice, we obtain the RG equation for $\frac{\delta^2 \Gamma_{BS}}{\delta \phi^2}$ as

$$D \frac{\delta^2 \Gamma_{BS}(\{c_i\}, \phi, v)}{\delta \phi^2} + 2\gamma_\phi \frac{\delta^2 \Gamma_{BS}(\{c_i\}, \phi, v)}{\delta \phi^2} + \gamma_\phi \phi \frac{\delta^3 \Gamma_{BS}(\{c_i\}, \phi, v)}{\delta \phi^3} = 0 \quad (21)$$

in both the R_ξ and the \overline{R}_ξ gauges. Then by applying D to Eq. (20) and by using Eq. (21), we obtain

$$0 = D \left[\frac{\delta^2 \Gamma_{BS}(\{c_i\}, \phi, v)}{\delta \phi^2} \right]_{\phi=0, p^2=-M^2} = (DM^2) \left[\frac{\delta^3 \Gamma_{BS}(\{c_i\}, \phi, v)}{\delta \phi^2 \delta(p^2)} \right]_{\phi=0, p^2=-M^2} - \left[2\gamma_\phi \frac{\delta^2 \Gamma_{BS}(\{c_i\}, \phi, v)}{\delta \phi^2} \right]_{\phi=0, p^2=-M^2} \quad (22)$$

Because the second term of above equation vanishes due to Eq. (20), we conclude that

$$DM^2 = 0 \quad (23)$$

and hence the pole mass is RG invariant in both cases. Finally, we will see that the pole mass of the Higgs field for both the R_ξ and the \overline{R}_ξ gauges are exactly the same up to one-loop. Because the one-loop pole mass for the \overline{R}_ξ gauge given in Eq. (41) of Ref. (8) is RG invariant, this shows that the one-loop pole mass for R_ξ is also RG invariant. In order to see this, let us note that up to one-loop, the pole mass is given by

$$M^2 = -2m^2 + [\Pi^{(1)}(p^2)]_{p^2=-2m^2}, \quad (24)$$

